Measurement of magnetic susceptibility of paramagnetic solutions by Quinck’s method

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Experiment completed: 30/08/2011
1 Objectives

- Determining the magnetic susceptibility $\chi$ of a given paramagnetic solution with a specific concentration.

- Calculating the mass susceptibility $\chi^\parallel$, molar susceptibility $\chi^\parallel\parallel$.

2 Theory

Paramagnetism is associated with the spins of the electrons. ‘Paramagnetism-the dipoles associated with the spins of the unpaired electrons experience a torque tending to line them up parallel to the field[1]’. Paramagnetic materials are drawn towards stronger magnetic fields as it reduces their potential energy. Understanding paramagnetism through classical ideas without knowledge of quantum mechanics is not very possible I feel. Unlike ferromagnetism, where the magnetic properties of the material are strikingly seen, paramagnetism and diamagnetism are comparatively much weaker phenomenon. The magnetization in paramagnetic and diamagnetic materials are sustained only as long as the external magnetic field is not zero and is not concerned with the whole history of magnetization but only on the current magnetic field, again unlike ferromagnets\(^1\) The magnetisation induced by the applied field is linear in field strength again unlike the non-linear behaviour of ferromagnets such as iron. The qualitative picture of diamagnetism can be given thus,

$$\frac{1}{4\pi\varepsilon_0} \frac{e^2}{R^2} = \frac{m_e v^2}{R}$$

\(^1\)No hysteresis curve for paramagnetic materials.
When the magnetic field is turned on, the magnetic force would change the velocity of the electron and the new equation would look like,

$$\frac{1}{4\pi\varepsilon_0} + ev' B = m_e \frac{v''^2}{R}$$

Thus, in this case the new speed is greater than the old speed.

$$ev' B = \frac{m_e}{R} (v'^2 - v^2) = \frac{m_e}{R} (v' + v)(v' - v)$$

If $\Delta v = v' - v$ is small,

$$\Delta v = \frac{eRB}{2m_e}$$

Change in the dipole moment of the electron due to the change in velocity is

$$\Delta M = -\frac{1}{2} e(\Delta v) R \hat{z} = -\frac{e^2 R^2}{4m_e} B$$

Thus the change in $M$ is opposite to the direction of $B$. Just as how Lenz law would say. In the absence of the magnetic field, all the magnetic dipoles in a diamagnetic atoms are oriented randomly. However, when we switch on the magnetic field, each atom acquires a dipole moment and the dipole moment of each atom is directed opposite to the magnetic field. Thus, there would now be a net dipole moment in the material. This is the mechanism of diamagnetism.

The diamagnetism mechanism applies for paramagnetic atoms too but the diamagnetism effect is overclouded by the paramagnetism effect as it is much stronger. The auxiliary field $\vec{H}$ is defined as,

$$\vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

(1)

$\vec{H}$ in magnetostatics plays an analogous role to $D$ in electrostatics[1]. $\vec{M}$ is the magnetic moment per unit volume of the magnetic material. For vacuum, it is zero.

In most of the materials the magnetization, $\vec{M}$ is proportional to the external

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2There will be a change in the magnetic field when the field is being turned on from 0 to the particular value $B$, this changing magnetic field produces an electric field that does the work in changing the velocity of the electron. As such, a magnetic field cannot do work.

3generally atoms with odd number of electrons.
magnetic field applied unless the applied field is too strong. So, we can introduce a constant here called $\chi$ which brings this linear relation.

$$\vec{M} = \chi \vec{H} \hspace{1cm} (2)$$

The linear relation is brought about by the fact that more the magnetic field, more fraction of the dipoles in a paramagnetic material get aligned in the direction of $\vec{H}$ as they have more torque across them enabling more number of dipoles to overcome the thermal effects which always keep trying to disorder them.

$$\vec{B} = \mu_o (H + m) \hspace{1cm} (3)$$

From (2), we can write this as,

$$\vec{B} = \mu_o (1 + \chi) \vec{H} \hspace{1cm} (4)$$

$$\vec{B} = \mu \vec{H} \hspace{1cm} (5)$$

where

$$\mu = \mu_o (1 + \chi) \hspace{1cm} (6)$$

$\mu$ is called the magnetic permeability of the material. $\mu_o$ is the permeability of free space. $\chi$ is a dimensionless quantity indicating the degree of magnetization of material in response to the magnetic field. Magnetic permeability also has a very similar meaning\(^4\), it too indicates the ability of the material to support the formation of magnetic field.

Here $\chi$ equals zero as there is no matter to magnetize.

Our apparatus looks like this.

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\(^4\mu_r\) (relative magnetic permeability) and $\chi$ just differ by the addition of constant 1. $\mu_r = 1 + \chi$. 

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Figure 1: Schematic diagram of Quinck’s method

Figure 2: The rise and fall of liquid in the narrow and the wide tube respectively.
The energy stored in magnetic fields is given as

\[ u = \frac{1}{2} \vec{H} \cdot \vec{H} = \frac{1}{2} |\vec{H}|^2 \]  

(7)

When the field is turned on, in our case, the solutions were paramagnetic, the solution will raise up by a height \( h \) into the magnetic field of the electromagnets. Thus the volume of solution replaces an equal amount of volume of air, and thus the potential energy of this region increases. The change in potential energy is given by,

\[ \Delta U = \frac{1}{2}(\mu - \mu_o)|\vec{H}|^2(\pi r^2h) \]  

(8)

The work done by the upward magnetic force \( F_m \) in raising the paramagnetic solution by an amount \( h \) is \( \Delta U = F_m h \).

\[ F_m = \frac{\Delta U}{h} = \frac{1}{2}(\mu - \mu_o)|\vec{H}|^2\pi r^2 \]  

(9)

This upward force has to be balanced by the downward force, for the solution at point A to remain stationary. It is balanced by the weight of the solution above A.

\[ F_g = mg = \rho(h + h')\pi r^2 g = \rho h(1 + \frac{r^2}{R^2})\pi r^2 g \]  

(10)

where \( r, R \) are the radius of the thinner and the wider tube respectively and \( \rho \) is the density of the solution. There will be a buoyant force too acting upward which is given by,

\[ F_b = \rho_o (h + h')\pi r^2 g = \rho_o h(1 + \frac{r^2}{R^2})\pi r^2 g \]  

(11)

where \( \rho_o \) is the density of the air. \( F_b \) acts in the same direction of \( F_m \), combining all the above three forces, we write

\[ F_m = F_g - F_b \]  

(12)

\[ \frac{1}{2}(\mu - \mu_o)H^2\pi r^2 = (\rho - \rho_o)h(1 + \frac{r^2}{R^2})\pi r^2 g \]  

(13)

From (6),

\[ \chi = 2g\mu_o(\rho - \rho_o)(1 + \frac{r^2}{R^2})\frac{h}{B^2} \]  

(14)
3 Apparatus

Electromagnet, power supply, Gauss meter, Hall probe, travelling microscope, Quinck’s tube, measuring flask, \( FeCl_3 \), \( MnCl_2 \), hair blower to dry the equipment, piknometer, weighing machine.

4 Experimental procedure

1. The density \((\rho)\) of the solution is determined using a pyknometer.

\[
\rho = \rho_{\text{water}} \frac{c - a}{b - a}
\]

where \(a\) is the weight of the pyknometer when it is empty and \(b\) is the weight of the pyknometer with distilled water and \(c\) is the weight of the pyknometer with the solution. We find density for every concentration of the solution.

2. The electromagnet coils in series to the power supply and ammeter is connected. The Hall probe (of the Gaussmeter), is used to measure the magnetic field. The Hall sensor probe inserted between the poles and for some current supply, we check at which position, the reading is coming out to be the maximum. At this position, the Gaussmeter is clamped and the current supply set to zero, we zero the reading of the Gaussmeter.

3. The magnetic field for varying currents (0 to 3.5 A) were recorded for current change in steps of 0.1 A.

4. The liquid solution is filled in the tube and the meniscus is kept in center of the two poles.

5. The microscope is focussed onto the miniscus and the zero reading is noted down.

6. The current through the magnet is changed till a noticeable change in the height of the solution is seen, and the new height of the meniscus is noted down.

7. The above step is repeated a few times for higher values of current.

8. The plot of \(\chi\) versus concentration is made.
5  Data, plot analysis and discussion

5.1  Calculations

Room temperature = 300K

- $r = 0.32$ cm
- $R = 1.23$ cm
- $\rho_o = 1000$ kg m$^{-3}$
- $\chi_| = \rho \chi$ is the mass susceptibility.
- $\chi_| = \rho \chi$ is the molar susceptibility.

The mean $\frac{h_B}{B^2}, \rho$ and the concentration is calculated for each concentration of the salt for each kind of salt and $\chi$ is calculated using the mean $\frac{h_B}{B^2}$ and is tabulated below. The diamagnetism of the water is accounted for while reporting the $\chi$ in the table below.

Table 1: Calculations for $MnCl_2.4H_2O$

<table>
<thead>
<tr>
<th>Amount (g)</th>
<th>Mean $\frac{h_B}{B^2} \times 10^{-3}$</th>
<th>$\rho$ kg m$^{-3}$</th>
<th>$\chi \times 10^{-5}$</th>
<th>concentration g mL$^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>2.89</td>
<td>1052</td>
<td>8.91</td>
<td>0.06</td>
</tr>
<tr>
<td>15</td>
<td>6.98</td>
<td>1080</td>
<td>20.74</td>
<td>0.095</td>
</tr>
<tr>
<td>20</td>
<td>1.29</td>
<td>1104</td>
<td>4.65</td>
<td>0.13</td>
</tr>
</tbody>
</table>

Table 2: Calculations for $FeCl_3.6H_2O$

| Amount (g) | Mean $\frac{h_B}{B^2} \times 10^{-3}$ | $\rho$ kg m$^{-3}$ | $\chi \times 10^{-5}$ | concentration g mL$^{-1}$ | $\chi_| $kg m$^{-3}$ | $\chi_| $kg g^2m^{-3}$ |
|------------|---------------------------------------|---------------------|-----------------------|--------------------------|---------------------|----------------------|
| 20         | 3.62                                  | 1105                | 11.42                 | 0.16                     | $12.6 \times 10^{-2}$ | 2.55                 |
| 25         | 4.13                                  | 1124                | 13.14                 | 0.2                      | $14.8 \times 10^{-2}$ | 2.99                 |
| 30         | 4.93                                  | 1168                | 16.06                 | 0.24                     | $18.8 \times 10^{-2}$ | 3.8                  |

We can observe that there is an order of difference between the diamagnetism of water and the paramagnetism of the solution indicating that the latter is more stronger than the former.
5.2 Some other points to discuss

- While checking for the position between the poles of the electromagnet where the Gaussmeter could detect the maximum magnetic field, we could notice that when we move the Hall sensor rapidly between the poles, there was a much bigger magnetic field recorded in the Gaussmeter. This is due to the eddy currents produced in the probe because of the change of magnetic flux through the probe.

- The effect of paramagnetism decreases on increase of temperature. The order which paramagnetism tries to establishing by lining up the dipole moments in the direction of the magnetic field is easily overcome by the thermal motions which try to derange the order. As more the temperature, the order will be more and more lost as temperature makes the thermal motions more pronounced. If this experiment was to be conducted at lower temperatures, then we could have observed a much better response in the paramagnetic salts in the solution raising up the capillary.

- The magnetic field produced in between the electromagnets in the order of thousands of Gauss, which is very big compared to the magnetic field of earth which is 0.3 Gauss. So, earth’s magnetic field is not a hindrance to the experiment being done.

- We have to take the reading of magnetic field recorded to the current flown through the electromagnet on both days the experiment is being done because of the hysteresis in the metal used for the electromagnet. Actually, it would be much better to take the magnetic field readings for each value of current as the experiment is running instead of taking everything at once in the beginning. However we must be cautious to extend the Hall sensor to the same exact position every time before recording the level of the miniscus. And also it has to be done fast.

- We discussed in the theory that the change in potential energy is equation.(8). However, if the volume of solution has occupied the volume of air, then the displaced volume of air should have occupied the displaced volume of solution somewhere else which is nowhere else but at the the other arm of the tube. So, here too there would be a change in magnetic potential energy which we don’t consider because we take the approximation that the magnetic field is produced only between the poles of the electromagnet and nowhere else.
• The $\frac{h}{B^2}$ ratio was coming to be much more constant for stronger concentrations.

• The gaussmeter uses a Hall sensor to measure the magnetic field. For a metal/semiconductor, which carries a current $\vec{I}$, if placed in a transverse magnetic field $\vec{B}$, an electric field $\vec{E}$ is induced in a direction perpendicular to both the current and the magnetic field. This phenomenon is known as the Hall effect\cite{4}. The electric field is produced due to the creation of an emf due to the force the magnetic field exerts on the charge carriers, which get pushed to the extremes of the material creating a potential difference between the ends of the material.

• We can also plot the hysteresis curve for the metal making our electromagnet. The negative current supply can be provided to the magnets by simply reversing the terminals of the power supply to the electromagnet.

• All the classical explanations given in the theory of this report fail on pursuing it far enough. Quantum mechanics is the only way through which it can properly be explained.\footnote{Section 34-6: Classical physics gives neither diamagnetism nor paramagnetism.}

• Both the salts have 5 unpaired electrons in the $d$ orbital.

6 Error analysis

6.1 Propagation of errors

I have done the error analysis only for $FeCl_{3.6}H_2O$.

$$\chi = 2g\mu_o(\rho - \rho_o)(1 + \frac{r^2}{R^2})\frac{h}{B^2}$$

Taking $\rho = \frac{m}{V}$, (we would get better results using the formula for $\rho$ given in the Procedure of this report.)

$$\frac{\Delta \chi}{\chi} = \frac{1}{\rho - \rho_o}[\frac{\Delta m}{m}\rho + \frac{\Delta m_o}{m_o}\rho_o] + \frac{\Delta h}{h} + \frac{2\Delta B}{B}$$

The first and third terms of the above expression give values which are two orders lesser than the value given by the middle term. And hence, they are ignored. The errors are tabulated below.
Table 3: Propagation errors

<table>
<thead>
<tr>
<th>Weight of solute (g)</th>
<th>$\frac{\Delta\chi}{\chi}$</th>
<th>$\Delta\chi \times 10^{-5}$</th>
<th>$\Delta\chi^\parallel \times 10^{-2} kg m^{-3}$</th>
<th>$\Delta\chi^\parallel \times kg^2 m^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.047</td>
<td>0.54</td>
<td>0.59</td>
<td>0.12</td>
</tr>
<tr>
<td>25</td>
<td>0.023</td>
<td>0.3</td>
<td>0.34</td>
<td>0.07</td>
</tr>
<tr>
<td>30</td>
<td>0.012</td>
<td>0.19</td>
<td>0.22</td>
<td>0.04</td>
</tr>
</tbody>
</table>

\[
\frac{\Delta\chi}{\chi} = \frac{\Delta\chi^\parallel}{\chi^\parallel} = \frac{\Delta\chi^\parallel}{\chi^\parallel}
\]

because all the quantities differ just by a multiplicative constant.

6.2 Sources of error

- We are assuming that the salt uniformly dissolves in distilled water. While keeping the capillary containing the solution upright between the electromagnetic poles, if the density of the solution is not uniform throughout, the heavier part of the solution (the part where the salt is dissolved more) would come down due to its density and the less denser solution would move up and the concentration of the solution we calculate may no longer match to the actual situation. This problem can be partially addressed by finishing the taking of the readings of the level of miniscus as fast as possible for all magnetic fields.

- But by taking the readings too fast, perhaps the effect of Eddy currents in the solution may become prominent. Now, the above two errors are cyclic.

- Because of hysteresis in the electromagnet, the magnetic field produced in between the electromagnet will be different each time for the same value of current. It was recorded that once the Gaussmeter was set to zero and the current changed till 3.5 A for recording the magnetic field and again when we reached back to zero current, the reading in the Gaussmeter was 127 gauss. We should make sure that the magnetic field is recorded in steps of 0.1 A from 0 to 3.5 continuously, without any travelling back and forth in between. The error mentioned here can perhaps be mitigated by providing a reverse current to the magnet by changing the terminals of the power supply and bringing the magnetic

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\(^6\)the distilled water has ions of the salt now which can conduct electricity
field again as close to zero as possible (idea by my lab partner Arka). However, we did not do the same while taking our readings.

- We assumed in our calculation that the number of water molecules per unit volume is not different in the solution from that in the pure water.

7 Conclusions and remarks

- As indicated in the manual, the required concentrations were asked to be prepared in 100mL quantities (Eg: 20g/100 mL). However, this much amount of solution is not needed for the experiment. As my lab partner, Arka suggested, we can as well obtain the same concentration by preparing the solution like 10g/50 mL saving chemicals and distilled water.

- \( \chi \) increased on increasing concentration of the salt which is expected (also it should be seen that at some point, a saturation point of the \( \chi \) should be attained). However, I could not find any theory which I could comprehend telling how this relationship between \( \chi \) and concentration is supposed to be. Thus, I have not drawn a best fit line for the got data in the graph.

- Further calculations for the salt of \( MnCl_2.4H_2O \), was not done as the \( \chi \) was not seen to increase on increase of concentration. The \( \chi \)s obtained for the solute concentration of 0.06g mL\(^{-1}\) and 0.095g mL\(^{-1}\) seem fine as \( \chi \) increased for increasing concentration. However, for 0.13g mL\(^{-1}\), the \( \frac{d\chi}{dT} \) somehow comes lower than the values obtained for the lower concentrations. I could not find out the reason for this anomaly.

- For \( FeCl_3.6H_2O \), the magnetic susceptibility, including the mass and molar susceptibility and their propagation errors are tabulated in the Tables (2 and 3).

8 Bibliography

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